## Harmonic Analysis Trial based on the Tidal Constituents list by TWG/IHO.

## 1. Introduction

In Japan, traditional 60 tidal constituents are used for tidal prediction of the major ports, which short period tides are derived from the 1 year harmonic analysis and long period tides are from several years.

Recently, tidal heights are shown in real-time base through the website of our JHOD, JMA and etc. In the site, the graphs of observed and predicted tidal heights and their deviation are shown. Higher harmonics are remarked in the deviation graph. Additionally, semidiurnal tides in some place are perhaps varied by season. The "standard tidal constituents list" by TWG/IHO includes the higher harmonic of M7 to M12, and M2A and M2B of seasonal change of major M2. Then we tried to apply the tidal constituents list to the harmonic analysis for several tidal records.

Some constituents in the list have almost same angular speed and unable to separate for several years, so we have selected 147 tidal constituents considering the traditionally well-known references.

## 2. Basic Tidal Equation and Harmonic Analysis by the direct Least Square Method.

Recently, tidal height are digitally observed by digital tidal gauge or electronic equipments, and recorded in shorter period such as 30 sec . Some tidal records are long continued over than 1year. We consider the trial applying the data in 6 min . interval for several years (2006-2009).

Basic tidal equation is same as traditional one;

$$
\mathrm{H}(\mathrm{t})=\sum \mathrm{fi} \cdot \mathrm{Ai} \cdot \cos \left\{\left(\mathrm{~V}_{0}+\mathrm{ui}\right)+\sigma \mathrm{i} \cdot \mathrm{t}-\kappa \mathrm{i}\right\}=\sum\{\mathrm{ACi} \cdot \cos \sigma \mathrm{i} \cdot \mathrm{t}+\mathrm{ASi} \cdot \cos \sigma \mathrm{i} \cdot \mathrm{t}\}
$$

fi,ui : Astronomical factors, $\mathrm{V}_{0 i}$ : Astronomical arguments, oi : Angular speed, $\mathrm{Ai} \cdot{ }_{K} \mathrm{i}$ : amplitude, phase lag. ACi, $\mathrm{ASi}: \cos$, sin components
Suffix i = tidal constituents $=60$, or $147($ M2A,M2B,,, and M7,M8,,, M12 )
fi,ui : varied with $(\mathrm{t})$, which was assumed constant during the analysis period in traditional harmonic analysis.
$(\mathrm{t}):$ time interval, 1 hour $\rightarrow 6 \mathrm{~min}$.

## 3. Results

Applying to the tidal height data of Hiroshima in 6 min . interval for 4 years, 2006-2009 (RDMDB/ NEAR-GOOS by JODC), (1) 147 and (2) 60 harmonic constants in the Table 1 are derived using the least square method directly. The previous harmonic constants (3) 60 were also listed in the Table, which were calculated by the T.I. method except the long period tides Sa and Ssa. In order to check each reliability of the HCs, tidal predictions are calculated for 1 year, 2010, using (1) , (2) and (3). Comparing with the observed data of 2010, the correlation coefficients are (1) 0.9944 , (2) 0.9942 , and (3) 0.9940 , and the standard deviations are (1) 9.13 , (2) 9.23 , and (3) 9.42 cm , respectively. Each amplitude and phase lag of major tidal constituent is marvelously same. Minor constituents added in (1) 147, are almost small except MA2 1.3 cm and 2 MO 51.1 cm . This result means each tidal prediction is not so different in a particular day as shown in Figure 1, but the each deviation from the observation is clearly different that (1) 147 HCs is able to predict the higher harmonic constituents compared to (2) and (3) 60 HCs .

Figure 1. Comparison of Tidal Predictions, Hiroshima 2010 (Obs.-Pred.).


## 4. Conclusion

This trial result shows the prediction by ordinary 60 HCs is practically enough because the correlation of Obs. and Pre. is more than 0.99 and S.D is less than 10 cm .

But for the more precise prediction, the higher harmonics of shallow water tides and seasonal fluctuation of major constituents such as MA2 for M2, should be included. In other trial case, the reliability of (1) 147 HCs is not better than (2) or (3) 60 HCs , in which reason the several day period fluctuation, perhaps non-tidal, may affect the tidal height. In conclusion, in order to improve the reliability of tidal prediction, considering the non-tidal components to separate well with tidal fluctuation, the proper tidal constituents set should be used representing well higher harmonics and fluctuation of major constituents.

## Reference

1) Standard Development of Tide Generating Potential.

IHB Circular Letter No.4-H, 1954, (Reprint from IHR May 1954), SP No. 40.
The Harmonic Development of the Tide-Generating Potential, by A.T. Doodson 1921.

60 Tidal constituents list :
"The Analysis of Tidal Observations", by A.T.Doodson, Publication of R.S. of London 1928
67 Tidal Constituents list:
"The Admiralty Method of Long Period Observations for Hamonic Tidal
Analysis", Admiralty Tidal Handbook No.1, by the Hydrographer of the Navy, N.P. 122(1)

147 Tidal Constituents list:
Selected from " Standard list of tidal constituents" by TG/IHO.
(unable to apply the whole constituents.*)

| Solution | Hiroshima 147 |  | Solutio Hiroshima 60 |  |  | 広島 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Corre. | 0.9944 |  | 0.994 |  |  | 0.994 |  |  |
| Obs-Pre. S.D. | 9.13 |  | 9.23 |  |  | 9.42 |  |  |
| Z0 | 325.1 |  | Z0 325.1 |  |  | Z0 | 200 |  |
| 1 Sa | 17.1 | 144.6 | Sa | 17.1 | 144.6 | Sa | 18.1 | 152.7 |
| 2 Ssa | 1.1 | 94.4 | Ssa | 1.1 | 94.4 | Ssa | 1 | 336.9 |
| 3 Mm | 0.9 | 96 | Mm | 0.9 | 96 | Mm | 1.8 | 238.9 |
| 4 MSf | 0.8 | 293.8 | MSf | 0.8 | 293.8 | MSf | 0.8 | 267.7 |
| 5 Mf | 1.1 | 92.1 | Mf | 1.1 | 92.1 | Mf | 1.2 | 206 |
| 6 2Q1 | 0.5 | 185.4 | 2Q1 | 0.5 | 185.4 | 2Q1 | 0.7 | 151.1 |
| 7 sigma1 | 1.1 | 245.1 | sigma1 | 1.1 | 245.1 | $\sigma 1$ | 0.8 | 239.5 |
| 8 Q1 | 4.6 | 182.8 | Q1 | 4.6 | 182.8 | Q1 | 4.4 | 187.1 |
| 9 rho1 | 1 | 162.9 | rho1 | 1 | 162.9 | م 1 | 1 | 167 |
| 1001 | 22.8 | 194.9 | O1 | 22.8 | 194.9 | 01 | 22.5 | 194.7 |
| 11 MP1 | 2.1 | 284.5 | MP1 | 2.1 | 284.4 | MP1 | 1.1 | 305.2 |
| 12 M 1 | 1.1 | 213.4 | M1 | 1.1 | 213.4 | M1 | 1.5 | 233.1 |
| 13 Khai1 | 0.2 | 161.5 |  | 0.2 | 161.6 | $\chi 1$ | 0.5 | 194.6 |
| 14 pai1 | 0.7 | 236.1 | Khai1 pai1 | 0.7 | 236 | $\pi 1$ | 0.7 | 223.8 |
| 15 P1 | 9.3 | 218.8 |  | 9.3 | 218.8 | P1 | 9.5 | 220.3 |
| 16 S1 | 1 | 100.2 | P1 S1 | 1 | 100.2 | S1 | 1.4 | 117.2 |
| 17 K1 | 31.3 | 216.9 | K1 | 31.3 | 216.9 | K1 | 31.2 | 217.1 |
| 18 psai1 | 0.5 | 250.8 | psai1 | 0.5 | 250.7 | $\psi 1$ | 0.4 | 179.4 |
| 19 fai1 | 0.4 | 172.2 | fai1 | 0.4 | 172.3 | $\phi 1$ | 0.5 | 189.4 |
| 20 theta1 | 0.4 | 253.8 | theta1 | 0.4 | 253.8 | $\theta 1$ | 0.3 | 262.2 |
| 21 J 1 | 1.7 | 256.3 | J1 | 1.7 | 256.3 | J1 | 1.6 | 257.7 |
| 22 SO1 | 1.5 | 5.6 | SO1 | 1.5 | 5.6 | S01 | 1.7 | 0 |
| 23001 | 1.2 | 283.9 | 001 | 1.2 | 283.9 | 001 | 0.9 | 321.5 |
| 24 OQ2 | 0.2 | 16.6 | OQ2 MNS2 | 0.2 | 17 | OQ2 | 0.5 | 226.5 |
| 25 MNS2 | 0.5 | 180 |  | 0.5 | 179.5 | MNS2 | 0.4 | 44.8 |
| 26 2MK2 | 0.5 | 142.5 | MNS2 |  |  |  |  |  |
| 27 2N2 | 2.1 | 266.7 | 2N2 myu2 | 2.1 | 267 | 2N2 | 2.3 | 250.6 |
| 28 myu2 | 0.9 | 18.7 |  | 0.9 | 18.5 | $\mu 2$ | 0.4 | 80.9 |
| 29 SNK2 | 0.3 | 3.7 | myu2 |  |  |  |  |  |
| 30 NA2 | 0.3 | 27.9 |  |  |  |  |  |  |
| 31 N2 | 17.8 | 265.9 | N2 | 17.8 | 266 | N2 | 17.8 | 266.3 |
| 32 NB2 | 0.6 | 202 |  |  |  |  |  |  |
| 33 nyu2 | 3.8 | 265.7 | $\begin{aligned} & \text { nyu2 } \\ & \text { OP2 } \end{aligned}$ | 3.9 | 265.7 | $\nu 2$ | 3.5 | 276.2 |
| 34 OP2 | 0.5 | 69 |  | 0.5 | 69.5 | OP2 | 0.7 | 187.9 |
| 35 MA2 | 1.3 | 200.5 |  |  |  |  |  |  |
| 36 M 2 | 102 | 277.8 | M2 | 102 | 277.8 | M2 | 102 | 278.3 |
| 37 MB2 | 0.3 | 324.9 |  |  |  |  |  |  |
| 38 MKS2 | 0.8 | 122.8 | MKS2 <br> lamda2 | 0.8 | 122.3 | MKS2 | 0.8 | 89.9 |
| 39 lamda2 | 1.6 | 297.7 |  | 1.6 | 297.8 | $\lambda 2$ | 1.9 | 308.6 |
| 40 L2 | 3.6 | 297.6 | L2 | 3.6 | 297.6 | L2 | 3.4 | 299.1 |
| 41 2SK2 | 0.3 | 42 |  |  |  |  |  |  |
| 42 T2 | 2.6 | 302.2 | T2 | 2.5 | 302.3 | T2 | 2.8 | 308.5 |
| 43 S2 | 42.4 | 307.8 | S2 | 42.4 | 307.8 | S2 | 41.9 | 307.8 |
| 44 R 2 | 0.4 | 306.3 | R2 | 0.5 | 305.7 | R2 | 0.7 | 315.5 |
| 45 K 2 | 12.2 | 303.4 | K2 | 12.2 | 303.4 | K2 | 12.1 | 305.1 |
| 46 MSN2 | 0.6 | 83.7 | MSN2 | 0.6 | 84.4 | MSN2 | 1 | 147.3 |
| $47 \mathrm{KJ2}$ | 0.7 | 129 | KJ2 | 0.7 | 129 | KJ2 | 0.6 | 136.4 |
| 48 2SM2 | 1.5 | 179 | 2SM2 | 1.5 | 179 | 2SM2 | 1.4 | 168.1 |
| 49 MQ3 | 0.1 | 143.6 | MQ3 | 0.1 | 144.3 | MO3 | 0.1 | 157.1 |
| 50 MO 3 | 0.3 | 150.9 |  |  |  |  |  |  |
| 51 2MP3 | 0.3 | 204.5 |  |  |  |  |  |  |
| * 52 MA3 | 0 | 143.9 |  |  |  |  |  |  |


| 53 M3 | 0.4 | 342.9 | M3 | 0.4 | 342.4 | M3 | 0.5 | 2.1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| * 54 MB3 | 0.1 | 233.5 |  |  |  |  |  |  |
| 55 MP3 | 0.4 | 224.9 | MP3 | 0.4 | 225.2 | SO3 | 0.3 | 64.7 |
| 56 MK3 | 0.3 | 181.4 | MK3 | 0.3 | 181.9 | MK3 | 0.1 | 29.8 |
| 57 2MQ3 | 0.1 | 238.8 |  |  |  |  |  |  |
| 58 SK3 | 0.4 | 226 | SK3 | 0.4 | 226.1 | SK3 | 0.3 | 210.1 |
| 59 3MK4 | 0.1 | 289.7 |  |  |  |  |  |  |
| 60 3MS4 | 0.2 | 320.5 |  |  |  |  |  |  |
| 61 MN4 | 0.5 | 28.4 | MN4 | 0.5 | 28.4 | MN4 | 0.6 | 34.9 |
| 62 Mnyu4 | 0.1 | 123 |  |  |  |  |  |  |
| 63 2MSK4 | 0.1 | 312.7 |  |  |  |  |  |  |
| 64 MA4 | 0.1 | 269 |  |  |  |  |  |  |
| 65 M4 | 1.2 | 42.5 | M4 | 1.2 | 42.5 | M4 | 1.8 | 39.2 |
| * 66 MB4 | 0.2 | 112.3 |  |  |  |  |  |  |
| 67 2MKS4 | 0.2 | 110.7 |  |  |  |  |  |  |
| 68 SN4 | 0.3 | 30.1 | SN4 | 0.2 | 30.3 | SN4 | 0.1 | 30.8 |
| 69 3MN4 | 0.1 | 306.4 |  |  |  |  |  |  |
| 70 MS4 | 1.3 | 63.4 | MS4 | 1.3 | 63.4 | MS4 | 1.8 | 72.5 |
| 71 MK4 | 0.5 | 47.7 | MK4 | 0.5 | 47.6 | MK4 | 0.7 | 60.9 |
| 72 2MSN4 | 0 | 221.4 |  |  |  |  |  |  |
| 73 S4 | 0.2 | 91.9 | S4 | 0.2 | 91.9 | S4 | 0.3 | 110.9 |
| 74 SK4 | 0.2 | 100.9 | SK4 | 0.2 | 100.9 | SK4 | 0.1 | 185.2 |
| 75 2MQ5 | 0.6 | 4.8 |  |  |  |  |  |  |
| 76 2MO5 | 1.1 | 23.8 |  |  |  |  |  |  |
| * 77 MA5 | 0.1 | 136.3 |  |  |  |  |  |  |
| 78 M5 | 0.4 | 92 |  |  |  |  |  |  |
| * 79 MB5 | 0.1 | 254.2 |  |  |  |  |  |  |
| 80 2MS5 | 0.1 | 3.3 |  |  |  |  |  |  |
| 81 3MQ5 | 0.2 | 214 |  |  |  |  |  |  |
| 82 MSK5 | 0.9 | 358 |  |  |  |  |  |  |
| 83 4MK6 | 0.1 | 280.3 |  |  |  |  |  |  |
| 84 4MS6 | 0.9 | 259.5 |  |  |  |  |  |  |
| 85 2MSNK6 | 0.1 | 297.8 |  |  |  |  |  |  |
| 86 2MN6 | 1.5 | 133.1 | 2MN6 | 1.5 | 133 | 2MN6 | 1.5 | 139.5 |
| 87 2Mnyu6 | 0.6 | 115.7 |  |  |  |  |  |  |
| 88 3MSK6 | 0.2 | 329.3 |  |  |  |  |  |  |
| 89 MA6 | 0.1 | 261 |  |  |  |  |  |  |
| 90 M6 | 3.1 | 143.7 | M6 | 3.1 | 143.6 | M6 | 3.3 | 149.6 |
| 91 MB6 | 0.1 | 239.9 |  |  |  |  |  |  |
| 92 MSN6 | 0.8 | 176.1 | MSN6 | 0.8 | 176.5 | MSN6 | 1 | 184.9 |
| 93 4MSN6 | 0.5 | 314.1 |  |  |  |  |  |  |
| 94 2MS6 | 4.2 | 175.8 | 2MS6 | 4.2 | 175.8 | 2MS6 | 4.5 | 183.3 |
| 95 2MK6 | 1.2 | 169.3 | 2MK6 | 1.2 | 169.4 | 2MK6 | 1.3 | 178.1 |
| 96 3MSN6 | 0.9 | 354 |  |  |  |  |  |  |
| 97 2SM6 | 1.2 | 223.1 | 2SM6 | 1.2 | 223.2 | 2SM6 | 0.9 | 220.2 |
| 98 MSK6 | 0.8 | 209.4 | MSK6 | 0.8 | 209.2 | MSK6 | 0.8 | 224.4 |
| 99 3MQ7 | 0.2 | 44.7 |  |  |  |  |  |  |
| 100 4MK7 | 0.3 | 60.6 |  |  |  |  |  |  |
| 101 MNSO7 | 0.2 | 84.2 |  |  |  |  |  |  |
| 102 MA7 | 0 | 190.5 |  |  |  |  |  |  |
| 103 M7 | 0 | 322.4 |  |  |  |  |  |  |
| 104 MB7 | 0 | 315.8 |  |  |  |  |  |  |
| 105 3MK7 | 0.2 | 83.8 |  |  |  |  |  |  |
| 106 MSKO7 | 0.2 | 311.7 |  |  |  |  |  |  |
| 107 2(MN)8 | 0.1 | 320.4 |  |  |  |  |  |  |
| 108 3MN8 | 0.1 | 302 |  |  |  |  |  |  |
| 109 MA8 | 0 | 93.2 |  |  |  |  |  |  |
| 110 M8 | 0.2 | 358.3 |  |  |  |  |  |  |
| 111 MB8 | 0 | 251.4 |  |  |  |  |  |  |


| 112 2MSN8 | 0.1 | 17.6 |  |
| :---: | :---: | :---: | :---: |
| 113 3MS8 | 0.4 | 37.5 |  |
| 114 4MSN8 | 0 | 339.7 |  |
| 115 2(MS)8 | 0.2 | 90.1 |  |
| 116 3MNO9 | 0.1 | 74.8 |  |
| 117 2(MN)K9 | 0 | 79.2 |  |
| 118 MA9 | 0 | 242 |  |
| 119 M9 | 0.1 | 134.9 |  |
| 120 MB9 | 0 | 7.8 |  |
| 121 4MK9 | 0.1 | 118.1 |  |
| 122 3MSK9 | 0.2 | 182.6 |  |
| 123 3M2N10 | 0.1 | 304.2 |  |
| 124 4MN10 | 0.1 | 350.6 |  |
| 125 MA10 | 0 | 94.8 |  |
| 126 M10 | 0.1 | 17.2 |  |
| 127 MB10 | 0 | 92.9 |  |
| 128 3MSN10 | 0.3 | 7.7 |  |
| 129 4MS10 | 0.5 | 23.5 |  |
| 130 2(MS)N11 | 0.1 | 94.3 |  |
| 131 3M2S10 | 0.4 | 61.7 |  |
| 132 3SMN10 | 0 | 201.6 |  |
| 133 3S2M10 | 0.1 | 112.8 |  |
| 134 MA11 | 0 | 243.9 |  |
| 135 M11 | 0 | 312.1 |  |
| 136 MB11 | 0 | 292 |  |
| 137 XMS11 | 0.1 | 63.9 |  |
| 138 4MSK11 | 0 | 2.5 |  |
| 139 5MN12 | 0 | 138.2 |  |
| 140 MA12 | 0 | 27.1 |  |
| 141 M12 | 0 | 146.5 |  |
| 142 MB12 | 0 | 62.3 |  |
| 143 4MSN12 | 0 | 141.1 |  |
| 144 5MS12 | 0.1 | 88.4 |  |
| 145 3M2SN1¢ | 0 | 212 |  |
| 146 4M2S12 | 0 | 168 |  |
| 147 3(MS)12 | 0 | 288.5 |  |

